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FRACTAL IMAGE COMPRESSION

Contract # N00014-91-C-0117
Quarterly Progress Report 8-12-91

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1. SUMMARY OF WORK. During the period May-July, 1991 we concentrated our development efforts in three areas, a survey of applications, algorithm development and theory. We are also continuing to evaluate hardware needs.

2. APPLICATIONS. We have begun to select specific applications for our compression algorithms. Our selection process is based on information gained through technical discussions with commercial groups selling image compression hardware and software, literature reviews and performance figures on commercially available software. The industry standard (JPEG) for image compression is based on the discrete cosine transform. We have chosen to evaluate the suitability of the fractal method in comparison with this transform. Several key differences are apparent.

2.1 Decompression speed. Fractal methods are slower in the image compression phase than methods based on the cosine transform (and will certainly remain so for the near future), but image decompression times based on fractals are comparable to commercially-available systems. Because we have barely begun to optimize computer code for the fractal methods, while JPEG has the advantage of six or seven years of intensive development, we believe that our improvement efforts will lead to a significant decompression speed advantage over the discrete cosine transform. Accordingly, we are targeting applications where image encoding will be performed by dedicated systems in a central facility and decoding can be performed in software at the users' facilities. We will discuss our approach to improving image decompression speeds below.

One particular application of our image compression would be in the distribution of image data on CD-ROMs. We have begun discussions with a CD-ROM distributor. In this case image decoding can be performed in software, on a PC or workstation. Also, several government and commercial applications seem feasible. One such is the storage and distribution of satellite image data.

2.2 Image quality. As always, image quality is largely subjective, and the ultimate judge of a compression system is the user of the imagery. Nevertheless, using the

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criterion of mean square error, fractal methods are fully competitive with commercial systems. Actually, we are able to obtain moderately better signal to noise ratios for a given degree of image compression for small images (256 by 256 pixels). One advantage of fractal compression methods is that the compression attainable at a given signal to noise ratio scales with image size, while commercial methods do not improve with image size. Thus, our compression ratios should be much better than competing techniques for large images. This gives us a potential advantage in high quality color imagery.

A second measure of image quality is artifacts. Both discrete transform methods and fractal methods show blocking artifacts at higher compression ratios, while the most apparent artifacts of fractal compression methods at low compression ratios tend to be more localized. We are attempting to overcome some of the artifact problems by investigating alternative tiling schemes. We will discuss the technical issues below.

2.3. Compression speed. Although compression speed is not the most important issue for our initial applications, the application of fractal techniques to a wide variety of image compression problems is dependent upon this factor. As we have described in previous reports of our work, our method is based on image tiling schemes, and the compression is dependent upon comparisons between tiles. We have begun, in a modest way to investigate new decoding schemes based on a generalization to the quadtree partitioning algorithm. We have also begun to study methods for reducing the computations in making the inter-tile comparisons. We will discuss some of this work below.

2.4. Color. The ability to compress color images is critical to our target application. We have applied our method to a YIQ scheme for compressing color images.

3. ALGORITHMS. Our system development work has been concentrated in the area of algorithms. We have begun to develop a Gaussian pivoting scheme for image decompression, a quadtree scheme for image compression, and a new method based on triangulation for image tiling.

3.1. Decompression. If we discretize to a pixel based image, the system of affine transforms for image compression may be written as a single matrix equation. In particular, if q is the vector representing the image (of dimension MN, where M and N are the numbers of pixels in an image row, and an image column, respectively) then the coding problem is to find an approximating x, a transform T, and a vector of intensity offsets b, so that

$$Tx + b = x, \qquad \text{and} \qquad \|x - q\| < \varepsilon$$

In this representation, T represents both intensity scalings and the affine correspondences, T and **b** code the image and may be taken as given in the decompression. A sufficient condition for existence of T and **b** satisfying these qualities is to make $\|Tq + b - q\|$ small. The transform **T** comes from correlated tiles, so if pixels in the domains of the affine transforms are associated with row labels and pixels in the ranges are associated with column labels, then T is a very sparse MN by MN matrix. Therefore, to decompress we must compute

$$x = (T - I)^{-1}b,$$

where I represents the identity matrix. T is sparse because the image is covered only once. Therefore, a pivoting scheme can be very efficient.

3.2. **Compression.** Many image processing algorithms have been embodied as transforms on quad trees. The basic idea is to decompose an image into four sub-images by subdividing each edge into two parts. The original image is assigned to the root of a tree and the sub-images to four branches. This process can be continued until we obtain a tree, whose leaves are individual pixels. In this context, our comparison process may be considered as matching one portion of the tree with another, possibly at a different branching level. We are investigating fast algorithms based on this concept.

The quad tree resulting from subdivision may loose symmetry properties if the subdivision is not into equal portions. This is certainly the case if an image dimension is not a power of 2. There may be some advantages with rectangular, rather than square subdivisions. We are investigating the trade-off.

3.3. **Tiling.** An affine transform in the plane is coded by two domain vectors and two range vectors. If we decompose the image into triangles, affine transforms moving one triangle into another may be conveniently described in terms of two sides of the range triangle and two sides of the domain triangle. We are investigating recursive schemes for partitioning the image into triangles. These schemes allow us to reduce the number of bits needed to specify a transform. Another advantage of our method is that the triangulation need not be regular. This permits us to adapt the triangulation to the image. We can triangulate more densely where the image is more detailed, e.g., where intensity variance in a given size of neighborhood is higher. We can also select triangles to follow image features, such as edges or local maxima or minima.

Selection of a system of affine transformations is based on calculating cross-correlations between domains and potential ranges. We are also investigating the possibility of using more efficient correlation schemes. One possibility, if the triangulation is well-adapted, is to reduce the number of pixels we use each triangle, that is, to calculate the cross-correlation on a sparser set of pixels. We will investigate the implementation and effects of this scheme during the next quarter.

4. THEORY. The introduction of tree structures and alternative tilings into our algorithmic work has led to the consideration of lattice structures as a possible foundation for the construction of exotic tilings. Recall that a lattice in R^n is a discrete subgroup generated by integral linear combinations of n independent basis vectors. One example is the hexagonal lattice H in R^2 generated by the basis vectors $e_1 = (0, 1)$ and $e_2 = (\sqrt{3}/2, 1/2)$. We can construct an exotic tiling based on the hexagonal lattice. To do this we note that the matrix

$$M = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$$

maps e_1 and e_2 into lattice vectors, and a complete set of representatives of H/MH consists of the centers of the seven hexagons closest to the origin. Put another way, MH is also a hexagonal lattice, with its lattice points coinciding with a subset of H. Seven lattice points of H are included in each of the Voroni cells (hexagons) of MH. This gives the first step in constructing a hept-tree (as in seven branches) whose leaves are the points of H. We can iterate this construction several times to obtain a regular hept-tree, whose branches at a given level represent an exotic partition of the plane. Note that this construction works in reverse to the quad-tree construction mentioned above. Nevertheless, this partition is self similar by construction and we may use the hept-tree as the basis of an image compression technique, in much the same way that the quad tree is used in our algorithms. Finding descendents or parents from a given node merely requires computing powers of M.

There is also a fractal version of the hexagonal construction, which involves M^{-1}. This latter construction is the same as discussed in Schroeder. This construction and other generalizations of the quad-tree construction might be of theoretical interest only, except that the boundaries of the large cells in the partition are fractal in appearance. Such boundaries would not contribute as strongly to blocking artifacts as rectangular partitions.

Another natural question to ask is whether the iterated affine transform method might be adapted to work with other self-similar tree structures which are constructed from lattices in the plane. Wavelet methods, for example, give such trees. Keeping the low frequency components of a wavelet transform and making inter-tile comparisons on the basis of these wavelet coefficients might be one means for reducing the computations needed to choose the range tile associated with a given domain.

Lawton and Resnikoff point out that this example may be generalized to other lattices, matrices M and to higher dimensions. They also supply some other examples of fractal tiling but do not state a tiling theorem for the finite discrete case. We give a proposition without proof which covers the computationally relevant cases.

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July 14, 1991

Dr. Michael Shlesinger
Office of Naval Research
800 North Quincy Street
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Re: Contract No. N00014-91-C-0117

Dear Dr. Shlesinger:

Enclosed is our First Quarterly Progress Report on the Fractal Image Compression research we are conducting for ONR, contract line item no. 0002, data item no. A001. As you can see, we are making significant progress using this promising new technology.

Please let us know if you have any questions.

Sincerely,

Dan Greenwood
President, NETROLOGIC, Inc.

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enclosure

We assume that $Z^k \subset R^k$ is a lattice, and that M is a k x k matrix with integer entries, $\det(M) \neq 0$. If order $\left(\frac{Z^k}{MZ^k}\right) = n$ we choose a set $\{P_1, P_2, \ldots P_n\}$ of representatives of the cosets of $\frac{Z^k}{MZ^k}$. We say that $S_0 \subset R^k$ is a tile of the lattice Z^k in R^k if S_0 is closed, $\bigcup_{p \in Z^k} = R^k$ and $S_0 + p \cap S_0 \cap q$ is of measure zero, when $p \neq q$.

Proposition (B. Bielefeld)

Let S_0 be a tile of R^k. Define $S_i = \bigcup_{i=1_0}^{n} M^{-1}(S_{i-1} + p_i)$. Then S_i is a tile of R^k for each $i \geq 1$.

The theorem on fractal tilings, as stated in Lawton and Resnikoff is incorrect. We believe that the following conjecture provides a correct method for constructing some fractal tilings from a general lattice in R^k.

Conjecture:

Assume that S_0 is a Voroni cell (tile) of the lattice Z^k. We are given a minimal set of coset representatives $\{p_1, p_2, \ldots p_n\}$ of $\frac{Z^k}{MZ^k}$, $n > 1$. If

1) $S' = \cup_i(S_0 + p_i)$ is connected

2) There is no additional lattice point in the convex hull of $\{p_1, p_2, \ldots p_n\}$

and

3) $0 < d_H(S_0, R^k - S') < 1$,

then $S_\infty = \lim_{n \to \infty} S_n$ exists and S_∞ is a tile of Z^k in R^n.

We actually want the interior of S_∞ to be nonempty. We are investigating alternative conditions to 2) and 3) for this to be true.

5. SYSTEM REQUIREMENTS. We are continuing our efforts to assess the systems requirements. Although our conclusions are preliminary, (see the section on applications, above) it is clear that there should be two initial objectives. First, we should develop the prototype as a research tool, which will provide a system for development and testing of the

compression algorithms we invent. As we develop new or improved algorithms they should be integrated into the prototype to expand its capabilities and to maintain a realistic working environment for our compression methods. Second, the development of a commercial product should be based on experience with the prototype and customer requirements.

One hardware subsystem currently under consideration is a digital signal processor (DSP). Several companies are marketing DSP boards designed to interface with one of the workstations we are considering. We are currently contacting several companies for further information.

6. REFERENCES.

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Statement A per telecon
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